

MATHEMATICS**Category-I (Q : 1 to 50)**

Category-I : Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, $\frac{1}{4}$ mark will be deducted.

01. Let $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)$. Then
- (A*) $x^2y_2 + xy_1 + n^2y = 0$ (B) $xy_2 - xy_1 + 2n^2y = 0$
 (C) $x^2y_2 + 3xy_1 - n^2y = 0$ (D) $xy_2 + 5xy_1 - 3y = 0$
02. Let $\varphi(x) = f(x) + f(1-x)$ and $f''(x) < 0$ in $[0, 1]$, then
- (A*) φ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2}, 1\right]$
 (B) φ is monotonic increasing in $\left[\frac{1}{2}, 1\right]$ and monotonic decreasing in $\left[0, \frac{1}{2}\right]$
 (C) φ is neither increasing nor decreasing in any sub interval of $[0, 1]$
 (D) φ is increasing in $[0, 1]$
03. $\int \frac{f(x)\varphi'(x) + \varphi(x)f'(x)}{(f(x)\varphi(x)+1)\sqrt{f(x)\varphi(x)-1}} dx =$
- (A) $\sin^{-1} \sqrt{\frac{f(x)}{\varphi(x)}} + c$ (B) $\cos^{-1} \sqrt{(f(x))^2 - (\varphi(x))^2} + c$
 (C*) $\sqrt{2} \tan^{-1} \sqrt{\frac{f(x)\varphi(x)-1}{2}} + c$ (D) $\sqrt{2} \tan^{-1} \sqrt{\frac{f(x)\varphi(x)+1}{2}} + c$
04. The value of $\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x dx$ is equal to
- (A) 27 (B) 54 (C) -54 (D*) 0
05. $\int_0^2 [x^2] dx$ is equal to
- (A) 1 (B*) $5 - \sqrt{2} - \sqrt{3}$ (C) $3 - \sqrt{2}$ (D) $\frac{8}{3}$
06. If the tangent to the curve $y^2 = x^3$ at (m^2, m^3) is also a normal to the curve at (M^2, M^3) , then the value of mM is
- (A) $-\frac{1}{9}$ (B) $-\frac{2}{9}$ (C) $-\frac{1}{3}$ (D*) $-\frac{4}{9}$
07. If $x^2 + y^2 = a^2$, then $\int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx =$
- (A) $2\pi a$ (B) πa (C*) $\frac{1}{2}\pi a$ (D) $\frac{1}{4}\pi a$
08. Let f , be a continuous function in $[0, 1]$, then $\lim_{n \rightarrow \infty} \sum_{j=0}^n \frac{1}{n} f\left(\frac{j}{n}\right)$ is

$$(A) \int_0^{\frac{1}{2}} f(x) dx$$

$$(B) \int_{\frac{1}{2}}^1 f(x) dx$$

$$(C^*) \int_0^1 f(x) dx$$

$$(D) \int_0^{\frac{1}{2}} f(x) dx$$

09. Let f be a differentiable function with $\lim_{x \rightarrow \infty} f(x) = 0$. If $y' + yf'(x) - f(x)f'(x) = 0$, $\lim_{x \rightarrow \infty} y(x) = 0$, then (where

$$y' \equiv \frac{dy}{dx})$$

$$(A) y + 1 = e^{f(x)} + f(x)$$

$$(B) y - 1 = e^{f(x)} + f(x)$$

$$(C^*) y + 1 = e^{-f(x)} + f(x)$$

$$(D) y - 1 = e^{-f(x)} + f(x)$$

10. If $x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x \right] dx$, $x > 0$ and $y(1) = \frac{\pi}{2}$ then the value of $\cos\left(\frac{y}{x}\right)$ is

$$(A) 1$$

$$(B^*) \log x$$

$$(C) e$$

$$(D) 0$$

11. Let $f(x) = 1 - \sqrt{x^2}$ where the square root is to be taken positive, then

$$(A) f \text{ has no extrema at } x = 0$$

$$(B) f \text{ has minima at } x = 0$$

$$(C^*) f \text{ has maxima at } x = 0$$

$$(D) f' \text{ exists at } 0$$

12. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $[a > 0]$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a is equal to

$$(A^*) 2$$

$$(B) \frac{1}{2}$$

$$(C) \frac{1}{4}$$

$$(D) 3$$

13. If a and b are arbitrary positive real numbers, then the least possible value of $\frac{6a}{5b} + \frac{10b}{3a}$ is

$$(A^*) 4$$

$$(B) \frac{6}{5}$$

$$(C) \frac{10}{3}$$

$$(D) \frac{68}{15}$$

14. If $2 \log(x+1) - \log(x^2-1) = \log 2$, then $x =$

$$(A^*) \text{ only } 3$$

$$(B) -1 \text{ and } 3$$

$$(C) \text{ only } -1$$

$$(D) 1 \text{ and } 3$$

15. The number of complex numbers p such that $|p| = 1$ and imaginary part of p^4 is 0, is

$$(A) 4$$

$$(B) 2$$

$$(C^*) 8$$

$$(D) \text{ infinitely many}$$

16. The equation $z\bar{z} + (2-3i)z + (2+3i)\bar{z} + 4 = 0$ represents a circle of radius

$$(A) 2 \text{ unit}$$

$$(B^*) 3 \text{ unit}$$

$$(C) 4 \text{ unit}$$

$$(D) 6 \text{ unit}$$

17. The expression $ax^2 + bx + c$ (a, b and c are real) has the same sign as that of a for all x if

$$(A) b^2 - 4ac > 0$$

$$(B) b^2 - 4ac \neq 0$$

$$(C^*) b^2 - 4ac \leq 0$$

$$(D) b \text{ and } c \text{ have the same sign as that of } a$$

18. In a 12 storied building, 3 persons enter a lift cabin. It is known that they will leave the lift at different floors. In how many ways can they do so if the lift does not stop at the second floor?

$$(A) 36$$

$$(B) 120$$

$$(C) 240$$

$$(D^*) 720$$

19. If the total number of m -element subsets of the set $A = \{a_1, a_2, \dots, a_n\}$ is k times the number of m element subsets containing a_4 , then n is

$$(A) (m-1)k$$

$$(B^*) mk$$

$$(C) (m+1)k$$

$$(D) (m+2)k$$

20. Let $I(n) = n^n, J(n) = 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)$ for all $(n > 1), n \in N$, then

$$(A^*) I(n) > J(n)$$

$$(B) I(n) < J(n)$$

$$(C) I(n) = J(n)$$

$$(D) I(n) = \frac{1}{2} J(n)$$

21. If $c_0, c_1, c_2, \dots, c_{15}$ are the Binomial co-efficients in the expansion of $(1+x)^{15}$, then the value of

$$\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \dots + 15\frac{c_{15}}{c_{14}}$$

- (A) 1240 (B*) 120 (C) 124 (D) 140

22. Let $A = \begin{bmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{bmatrix}$ and $\det A = 5$, then

- (A) $t = 1$ (B) $t = 2$ (C) $t = -1$ (D*) $t = -2$

23. Let $A = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix}$. The value of x for which the matrix A is not invertible is

- (A) 6 (B) 12 (C*) 3 (D) 2

24. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2×2 real matrix with $\det A = 1$. If the equation $\det (A - \lambda I_2) = 0$ has imaginary roots

(I_2 be the Identity matrix of order 2), then

- (A*) $(a+d)^2 < 4$ (B) $(a+d)^2 = 4$ (C) $(a+d)^2 > 4$ (D) $(a+d)^2 = 16$

25. If $\begin{vmatrix} a^2 & bc & c^2 + ac \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix} = ka^2b^2c^2$, then $k =$

- (A) 2 (B) -2 (C) -4 (D*) 4

26. If $f : S \rightarrow \mathbb{R}$ where S is the set of all non-singular matrices of order 2 over \mathbb{R} and $f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$,

then

- (A) f is bijective mapping. (B) f is one-one but not onto.
(C) f is onto but not one-one. (D*) f is neither one-one nor onto.

27. Let the relation ρ be defined on \mathbb{R} by a ρ b holds if and only if $a - b$ is zero or irrational, then

- (A) ρ is equivalence relation.
(B*) ρ is reflexive & symmetric but is not transitive.
(C) ρ is reflexive and transitive but is not symmetric.
(D) ρ is reflexive only.

28. The unit vector in ZOY plane, making angles 45° and 60° respectively with $\vec{\alpha} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{j} - \hat{k}$ is

- (A) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ (B*) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$ (C) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$ (D) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$

29. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game. What is the probability that A wins if A begins?

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{7}{12}$ (D*) $\frac{8}{15}$

30. A rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds he must fire to have more than 50% chance of hitting it at least once, is

- (A) 5 (B*) 7 (C) 9 (D) 11

31. $\cos(2x + 7) = a(2 - \sin x)$ can have a real solution for
 (A) all real values of a (B) $a \in [2, 6]$
 (C) $a \in (-\infty, 2) \setminus \{0\}$ (D) $a \in (0, \infty)$
 No option matching
32. The differential equation of the family of curves $y = e^x(A \cos x + B \sin x)$ where A, B are arbitrary constants is
 (A) $\frac{d^2y}{dx^2} - 9x = 13$ (B*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
 (C) $\frac{d^2y}{dx^2} + 3y = 4$ (D) $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - xy = 0$
33. The equation $r \cos\left(\theta - \frac{\pi}{3}\right) = 2$ represents
 (A) a circle (B) a parabola (C) an ellipse (D*) a straight line
34. The locus of the centre of the circles which touch both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4ax$ externally is
 (A) a circle (B) a parabola (C) an ellipse (D*) a hyperbola
35. Let each of the equations $x^2 + 2xy + ay^2 = 0$ & $ax^2 + 2xy + y^2 = 0$ represent two straight lines passing through the origin. If they have a common line, then the other two lines are given by
 (A) $x - y = 0, x - 3y = 0$ (B*) $x + 3y = 0, 3x + y = 0$
 (C) $3x + y = 0, 3x - y = 0$ (D) $(3x - 2y) = 0, x + y = 0$
36. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at P and Q respectively. The point O divides the segment PQ in the ratio
 (A) 1 : 2 (B*) 3 : 4 (C) 2 : 1 (D) 4 : 3
37. Area in the first quadrant between the ellipses $x^2 + 2y^2 = a^2$ and $2x^2 + y^2 = a^2$ is
 (A*) $\frac{a^2}{\sqrt{2}} \tan^{-1} \frac{1}{\sqrt{2}}$ (B) $\frac{3a^2}{4} \tan^{-1} \frac{1}{2}$ (C) $\frac{5a^2}{2} \sin^{-1} \frac{1}{2}$ (D) $\frac{9\pi a^2}{2}$
38. The equation of circle of radius $\sqrt{17}$ unit, with centre on the positive side, of x-axis and through the point (0, 1) is
 (A*) $x^2 + y^2 - 8x - 1 = 0$ (B) $x^2 + y^2 + 8x - 1 = 0$
 (C) $x^2 + y^2 - 9y + 1 = 0$ (D) $2x^2 + 2y^2 - 3x + 2y = 4$
39. The length of the chord of the parabola $y^2 = 4ax$ ($a > 0$) which passes through the vertex and makes an acute angle α with the axis of the parabola is
 (A) $\pm 4a \cot \alpha \operatorname{cosec} \alpha$ (B*) $4a \cot \alpha \operatorname{cosec} \alpha$
 (C) $-4a \cot \alpha \operatorname{cosec} \alpha$ (D) $4a \operatorname{cosec}^2 \alpha$
40. A double ordinate PQ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is such that $\triangle OPQ$ is equilateral, O being the centre of the hyperbola. Then the eccentricity e satisfies the relation
 (A) $1 < e < \frac{2}{\sqrt{3}}$ (B) $e = \frac{2}{\sqrt{3}}$ (C) $e = \frac{\sqrt{3}}{2}$ (D*) $e > \frac{2}{\sqrt{3}}$
41. If B and B' are the ends of minor axis and S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then the area of the

rhombus SBS'B' will be

- (A) 12 sq. unit (B) 48 sq. unit (C*) 24 sq. unit (D) 36 sq. unit

42. The equation of the latus rectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$. Then the length of the latus rectum is

- (A) $4\sqrt{2}$ unit (B) $2\sqrt{2}$ unit (C) 8 unit (D*) $8\sqrt{2}$ unit

43. The equation of the plane through the point $(2, -1, -3)$ and parallel to the lines $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-4}$ and

$$\frac{x}{2} = \frac{y-1}{-3} = \frac{z-2}{2} \text{ is}$$

- (A) $8x + 14y + 13z + 37 = 0$ (B) $8x - 14y - 13z - 37 = 0$
(C) $8x - 14y - 13z + 37 = 0$ (D) $8x - 14y + 13z + 37 = 0$

No option matching

44. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

- (A) $\frac{2\sqrt{3}}{5}$ (B*) $\frac{\sqrt{2}}{10}$ (C) $\frac{4}{5\sqrt{2}}$ (D) $\frac{\sqrt{5}}{6}$

45. Let $f(x) = \sin x + \cos ax$ be periodic function. Then

- (A) 'a' is any real number. (B) 'a' is any irrational number.
(C*) 'a' is rational number. (D) $a = 0$

46. The domain of $f(x) = \sqrt{\left(\frac{1}{\sqrt{x}} - \sqrt{(x+1)}\right)}$ is

- (A) $x > -1$ (B) $(-1, \infty) \setminus \{0\}$ (C*) $\left(0, \frac{\sqrt{5}-1}{2}\right)$ (D) $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

47. Let $y = f(x) = 2x^2 - 3x + 2$. The differential of y when x changes from 2 to 1.99 is

- (A) 0.01 (B) 0.18 (C*) -0.05 (D) 0.07

48. If $\lim_{x \rightarrow 0} \left(\frac{1+cx}{1-cx}\right)^{1/x} = 4$, then $\lim_{x \rightarrow 0} \left(\frac{1+2cx}{1-2cx}\right)^{1/x}$ is

- (A) 2 (B) 4 (C*) 16 (D) 64

49. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be twice continuously differentiable (or f'' exists and is continuous) such that $f(0) = f(1) = f'(0) = 0$. Then

- (A*) $f''(c) = 0$ for some $c \in \mathbb{R}$ (B) there is no point for which $f''(x) = 0$
(C) at all points $f''(x) > 0$ (D) at all points $f''(x) < 0$

50. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12$. Then

- (A) $f(x)$ has 13 non-zero real roots.
(B*) $f(x)$ has exactly one real root.
(C) $f(x)$ has exactly one pair of imaginary roots.
(D) $f(x)$ has no real root.

51. The area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$ is

(A) $\frac{\pi^2}{2}$

(B) $\frac{\pi}{4}$

(C*) $\frac{\pi}{4} - \frac{1}{2}$

(D) $\frac{\pi^2}{3}$

52. In open interval $\left(0, \frac{\pi}{2}\right)$,

(A) $\cos x + x \sin x < 1$

(B*) $\cos x + x \sin x > 1$

(C) no specific order relation can be ascertained between $\cos x + x \sin x$ and 1

(D) $\cos x + x \sin x < \frac{1}{2}$

53. If the line $y = x$ is a tangent to the parabola $y = ax^2 + bx + c$ at the point (1, 1) and the curve passes through (-1, 0), then

(A) $a = b = -1, c = 3$

(B) $a = b = \frac{1}{2}, c = 0$

(C*) $a = c = \frac{1}{4}, b = \frac{1}{2}$

(D) $a = 0, b = c = \frac{1}{2}$

54. If the vectors $\vec{\alpha} = \hat{i} + a\hat{j} + a^2\hat{k}$, $\vec{\beta} = \hat{i} + b\hat{j} + b^2\hat{k}$ and $\vec{\gamma} = \hat{i} + c\hat{j} + c^2\hat{k}$ are three non-coplanar vectors and

$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is

(A) 1

(B) 0

(C*) -1

(D) 2

55. Let z_1 and z_2 be two imaginary roots of $z^2 + pz + q = 0$, where p and q are real. The points z_1, z_2 and origin form an equilateral triangle if

(A) $p^2 > 3q$

(B) $p^2 < 3q$

(C*) $p^2 = 3q$

(D) $p^2 = q$

56. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$ [a, b, c, d, are all real], then $P(x).Q(x) = 0$ has

(A*) at least two real roots

(B) Two real roots

(C) four real roots

(D) no real root

57. Let $A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$ & $f : A \rightarrow A$ be a mapping defined by $f(x) = x|x|$. Then f is

(A) injective but not surjective

(B) surjective but not injective

(C) Neither injective nor surjective

(D*) bijective

58. Let $f(x) = \sqrt{x^2 - 3x + 2}$ and $g(x) = \sqrt{x}$ be two given functions. If S be the domain of $f \circ g$ and T be the domain of $g \circ f$, then

(A) $S = T$

(B) $S \cap T = \emptyset$

(C) $S \cap T$ is a singleton

(D*) $S \cap T$ is an interval.

59. Let ρ_1 and ρ_2 be two equivalence relations defined on a non-void set S. Then

(A) both $\rho_1 \cap \rho_2$ and $\rho_1 \cup \rho_2$ are equivalence relations.

(B*) $\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so.

(C) $\rho_1 \cup \rho_2$ is equivalence relation but $\rho_1 \cap \rho_2$ is not so.

68. The equation $x^{(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5} = 3\sqrt{3}$ has
 (A*) at least one real root (B) exactly one real root
 (C*) exactly one irrational root (D) complex roots
69. In a certain test, there are n questions. In this test 2^{n-1} students gave wrong answers to at least i questions, where $i = 1, 2, \dots, n$. If the total number of wrong answers given is 2047, then n is equal to
 (A) 10 (B*) 11 (C) 12 (D) 13
70. A and B are independent events. The probability that both A and B occur is $\frac{1}{20}$ and the probability that neither of them occurs is $\frac{3}{5}$. The probability of occurrence of A is
 (A) $\frac{1}{2}$ (B) $\frac{1}{10}$ (C*) $\frac{1}{4}$ (D*) $\frac{1}{5}$
71. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is -1 is
 (A*) $\frac{x}{2} - \frac{y}{3} = 1$ (B*) $\frac{x}{-2} + \frac{y}{1} = 1$ (C) $-\frac{x}{3} + \frac{y}{2} = 1$ (D) $\frac{x}{1} - \frac{y}{2} = 1$
72. Consider a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$ at any point. The locus of the midpoint of the portion intercepted between the axes is
 (A) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ (B) $\frac{x^2}{4} + \frac{y^2}{2} = 1$ (C) $\frac{1}{3x^2} + \frac{1}{4y^2} = 1$ (D*) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$
73. Let $y = \frac{x^2}{(x+1)^2(x+2)}$. Then $\frac{d^2y}{dx^2}$ is
 (A*) $2 \left[\frac{3}{(x+1)^4} - \frac{3}{(x+1)^3} + \frac{4}{(x+2)^3} \right]$ (B) $3 \left[\frac{2}{(x+1)^3} + \frac{4}{(x+1)^2} - \frac{5}{(x+2)^3} \right]$
 (C) $\frac{6}{(x+1)^3} - \frac{4}{(x+1)^2} + \frac{3}{(x+1)^3}$ (D) $\frac{7}{(x+1)^3} - \frac{3}{(x+1)^2} + \frac{2}{(x+1)^3}$
74. Let $f(x) = \frac{1}{3} x \sin x - (1 - \cos x)$. The smallest positive integer k such that $\lim_{x \rightarrow 0} \frac{f(x)}{x^k} \neq 0$ is
 (A) 4 (B) 3 (C*) 2 (D) 1
75. Tangent is drawn at any point $P(x, y)$ on a curve, which passes through (1, 1). The tangent cuts X-axis and Y-axis at A and B respectively. If $AP : BP = 3 : 1$, then
 (A*) the differential equation of the curve is $3x \frac{dy}{dx} + y = 0$
 (B) the differential equation of the curve is $3x \frac{dy}{dx} - y = 0$
 (C*) the curve passes through $\left(\frac{1}{8}, 2\right)$
 (D) the normal at (1, 1) is $x + 3y = 4$